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Introduction to C++: Monte Carlo Project
Assignment 1

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1 Introduction

In the 1800s, the success of an army depended on its ability to fire quickly and accurately. In the era before drones and precision weaponry, mathematicians and engineers were employed to optimise the use of canons, muskets, and rifles. Soldiers typically marched together in columns and fired single shots row-by-row. With muskets and rifles especially exhibiting limited range, the enemy often had to be within 100m before fire could be considered accurate.

Muskets were capable of firing one bullet at a time, and had to be reloaded after every shot. In addition to the ability of the man holding the gun to stay steady, the weather could have a large effect on the accuracy of the bullet. Your task will be to run some simple Monte Carlo simulations to find an optimal strategy in a few different circumstances.

Figure 1: Re-enacters of the Battle of Waterloo. Soldiers are formed in ranks, armed with muskets.

The below tasks are the outline of a reasonably involved project. Do not worry if you do not make it as far as you would like through the bullet points, the important thing is that you have a chance to work with C++ on a project and gain an appreciation for the uses of Monte Carlo. That said, the more you put into it the more you get out, so please try to get as far as you can.

2 Main Project

And what makes a good soldier, Sharpe?
The ability to fire three rounds a minute in any weather, sir.

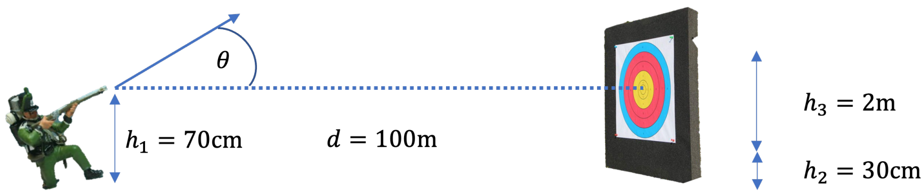

Figure 2: An illustration of the setup described in the first step.

A soldier is firing at a target 100m away. The bullet leaves his musket 70cm above the ground. The target itself is a circle of diameter 2m, the base of which is 30cm above the ground. The bullet has a speed of 450m/s.

The soldier can fire three times per minute, and tries to fire at an angle of his choosing, θ_{c}. Due to his shaky hand and imperfections in the gun's manufacture, there is an uncertainty added to this, θ_{u}, such that

$$\theta = \theta_{\mathrm{c}} + \theta_{\mathrm{u}}. \tag{1}$$

The soldier is a good shot, so θ_{u} is sampled from a normal distribution described by $\mu = 0°$ and $\sigma = 3°$.

- Numerically calculate the optimal angle (to four s.f.) that would be required to hit a bullseye. Tip: first compute the flight time for a given angle (horizontal SUVAT), and then determine the final vertical displacement.

- In one trial, the soldier fires repeatedly for five minutes. He chooses the optimal angle to hit the bullseye to be his θ_{c}. Simulate 1000 trials and construct a histogram showing the number of times he hits the target. What is the mean and standard deviation of these trials? Repeat for 10,000 and 100,000 trials.

- Highlight some key findings to be presented, e.g. in what percentage of trials is the soldier hitting the target at least 50% of the time?

SUVAT equations:
$s = \frac{u+v}{2}t$
$v = u + at$
$v^2 = u^2 + 2as$
$s = ut + \frac{1}{2}t^2$
$s = vt - \frac{1}{2}at^2$

3 Extension

Now that's soldiering

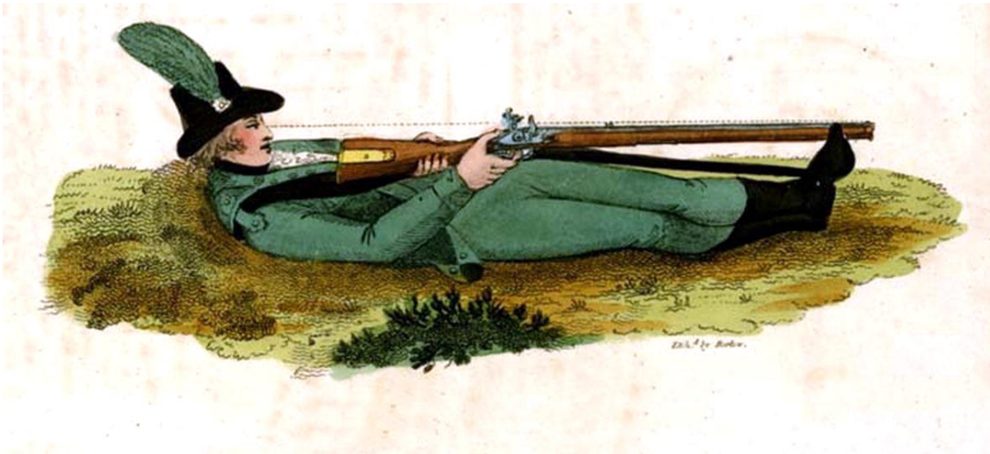

Figure 3: A depiction of a Baker rifle in action.

The soldier switches his musket for a rifle, which has greater accuracy ($\sigma = 1°$) but must be fired while lying down, so the distance of the muzzle above the ground, h_1, is now 30cm. Because he is lying down it takes him longer to reload, so the rate of fire is two times per minute. The construction of the rifle speeds up the bullet to 600m/s.

- Repeat the above tests for the rifle. What is the mean number of times he hits the target, assuming θ_c is the optimal angle to hit the bullseye? What is the standard deviation, and how does this compare with the musket?

The soldier recognises that both guns have their merits, with muskets being generally more effective at close range due to their higher rate of fire, and rifles being better at longer distances. He wants to know the distance at which muskets and rifles are equally effective.

- Run trials for the rifle and musket where the distance to the target, d is variable. For each weapon, plot the mean and standard deviation of the number of hits as a function of d. Find the distance at which they are equally effective. For each distance you will need to recompute θ_c.